Basic Trigonometry

Mane Shahinyan

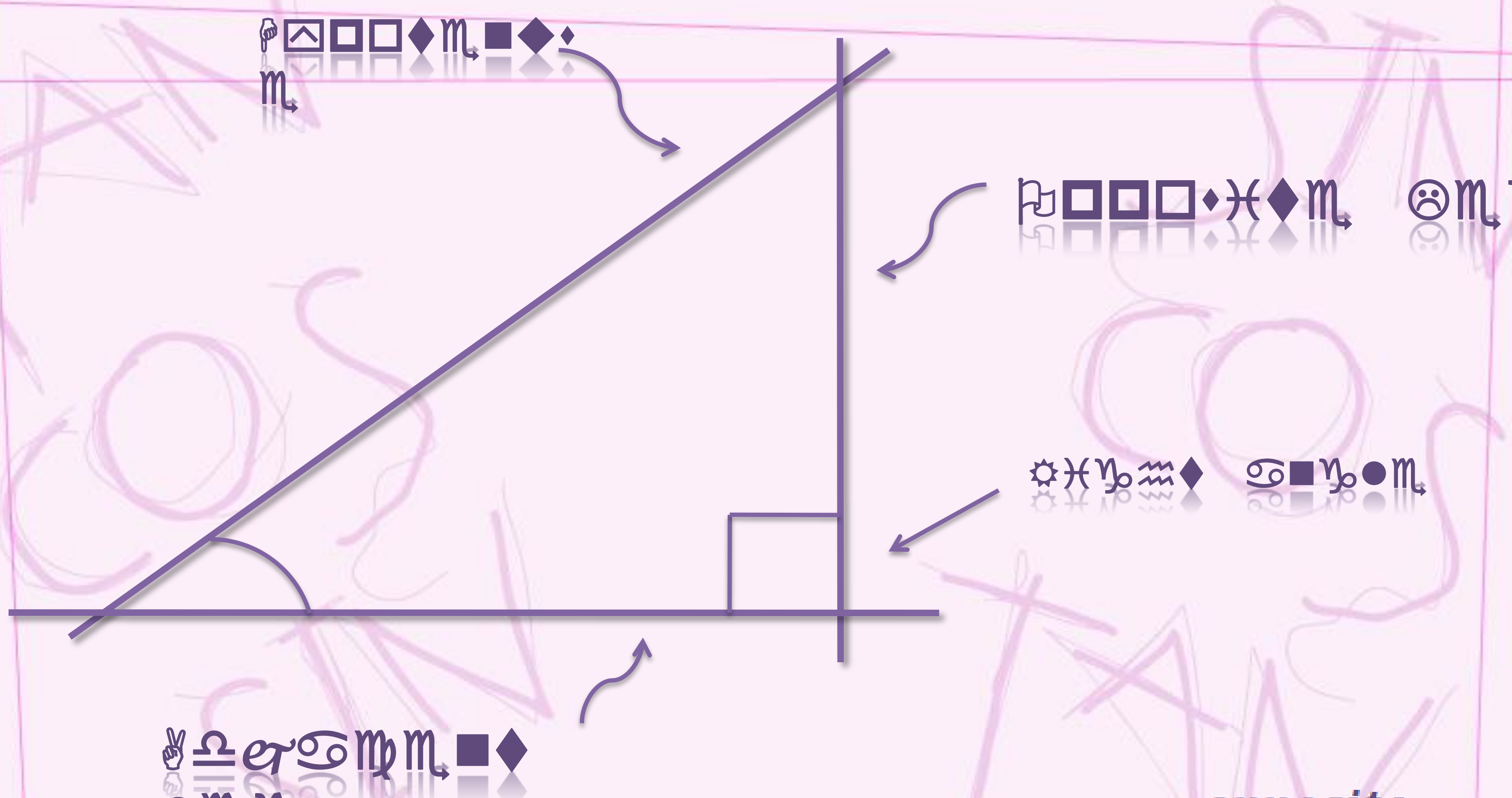
Trigonometry

- Trigonometry is derived from Greek words *trigonon* (three angles) and *metron* (measure).

- Greek astronomer Hipparcus is the father of trigonometry.

- Trigonometry deals with triangles, particularly triangles in a plane where one angle of the triangle is 90 degrees.

- Trigonometry deals with the relationships between the sides and the angles of triangles, that is, on the trigonometric functions.



$$\sin = \frac{opposite}{hypotenuse} \qquad \cos = \frac{adjacent}{hypotenuse} \qquad \tan = \frac{opposite}{adjacent}$$

$$cosecant(cosec) = \frac{hypotenuse}{opposite} \qquad secant(sec) = \frac{hypotenuse}{adjacent} \qquad cotangent(cot) = \frac{adjacent}{opposite}$$

Values of trigonometric functions of Angle α

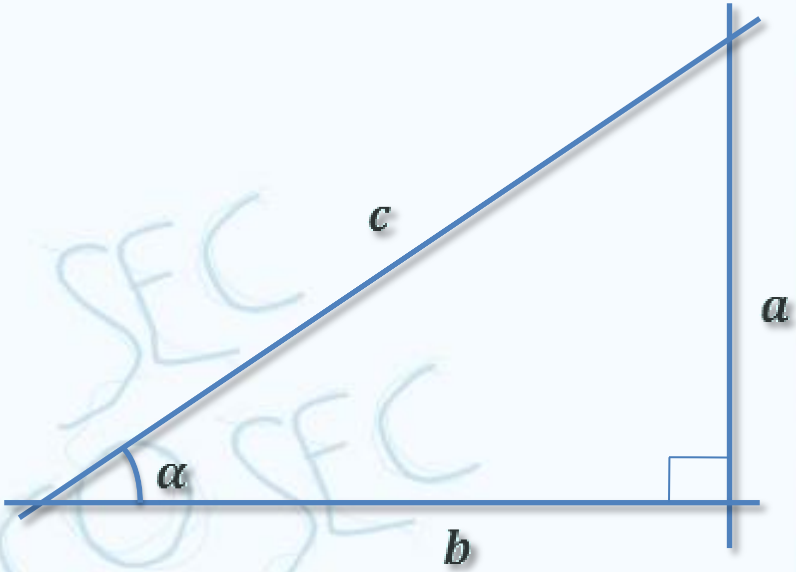
$$\sin(\alpha) = \frac{a}{c}$$

$$\cos(\alpha) = \frac{b}{c}$$

$$\tan(\alpha) = \frac{a}{b} = \frac{\sin(\alpha)}{\cos(\alpha)}$$

$$\cot(\alpha) = \frac{b}{a} = \frac{\cos(\alpha)}{\sin(\alpha)}$$

$$\sec(\alpha) = \frac{c}{b} = \frac{1}{\cos(\alpha)}$$



$$\operatorname{cosec}(\alpha) = \frac{c}{a} = \frac{1}{\sin(\alpha)}$$

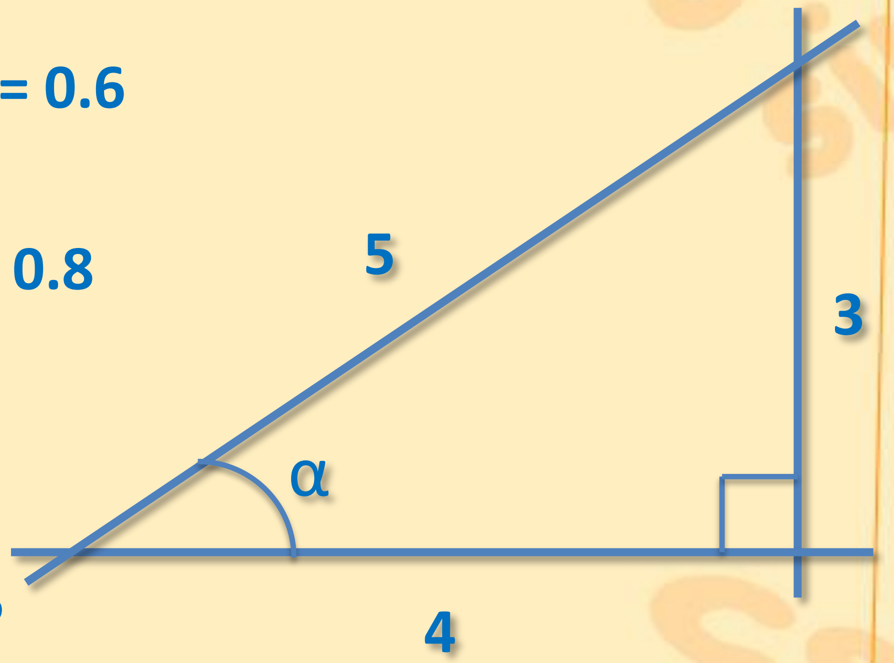
Try yourself

- **Find sinα-?**

Sinα = 3/5 = 0.6

- **Find cosα-?**

Cosα = 4/5 = 0.8



- **Find tanα -?**

Tanα = 3/4 =0.75 or
= sinα/cosα=0.6/0.8= 0.75

Secα - ?

Cosecα-?

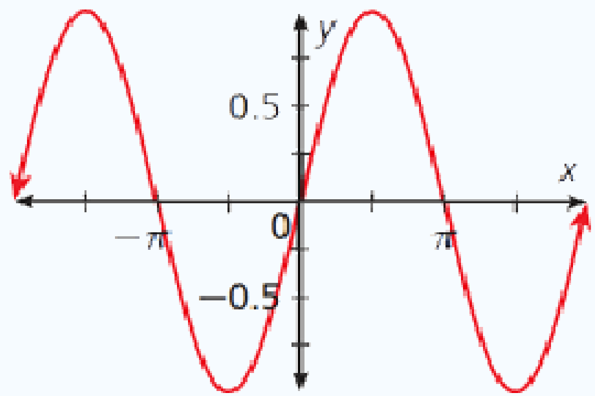
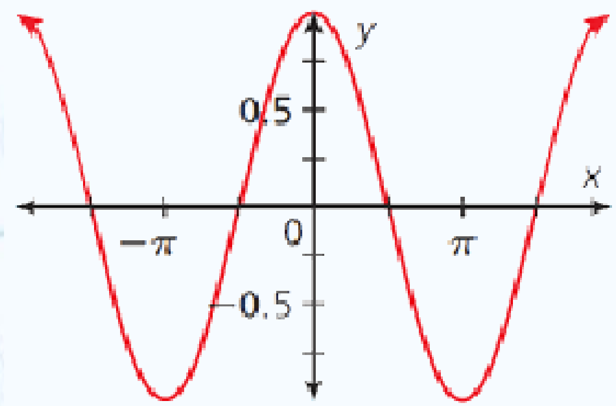
Secα = 1/cosα= 1/(4/5)= 5/4

Cscα = 1/sinα = 1/(3/5) = 5/3

Cotα-?

Cotα=cosα/sinα= 0.8/0.6=1.3(3)

Characteristics of the Graphs of Sine and Cosine

Function	Y=sinx	Y=cosx
Graph (sinusoidal)		
Domain	is all real numbers, $(-\infty;\infty)$	is all real numbers, $(-\infty;\infty)$
Range	is the interval [-1, 1]	is the interval [-1, 1]
Period	2π	2π

Pythagorean Identity

- For any angle $\theta, \ cos^2(\theta) + sin^2(\theta) = 1$

Proof:

On a circle of radius r and angle of θ, we can find the coordinates of the point (x,y).

$\boldsymbol{x = rcos(\theta)}$ & $\boldsymbol{y = rsin(\theta)}$

Equation of circle is $x^2 + y^2 = r^2$ (substituting the relations above)

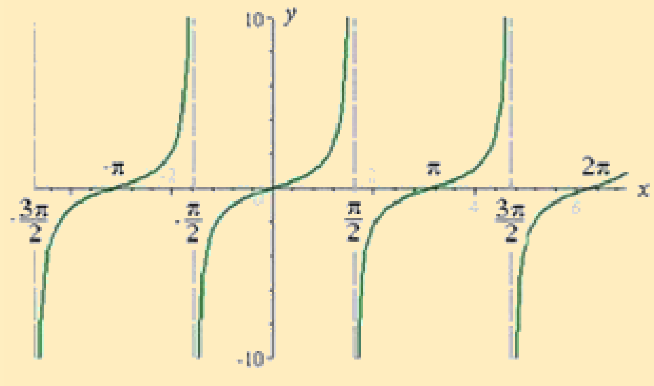
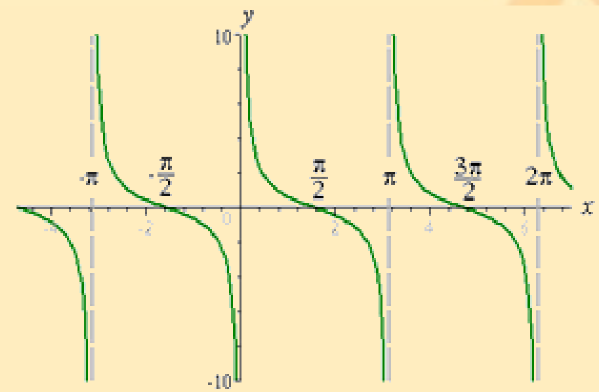
$(\boldsymbol{rcos(\theta)})^2 + (\boldsymbol{rsin(\theta)})^2 = \boldsymbol{r}^2$ Simplifying

$r^2(\cos(\theta))^2 + r^2(\sin(\theta))^2 = r^2$ Dividing by r^2

$(\cos(\theta))^2 + (\sin(\theta))^2 = 1$ Or using the shorthand notation

$$cos^2(\theta) + sin^2(\theta) = 1$$

Characteristics of the Graph of tan and cot

Function	Y=tanx	Y=cotx
Graph		
Domain	The tangent function is undefined at π/2. $\theta \neq \frac{\pi}{2} + k\pi, where\ k\ is\ an\ integer$	$\theta \neq k\pi, where\ k\ is\ an\ integer$
Range	Is all real numbers $(-\infty, \infty)$	Is all real numbers $(-\infty, \infty)$
Period	π	π

The signs of the trig functions

	90º	
	sin +ve cos –ve tan –ve	sin +ve cos +ve tan +ve	
180º			0º, 360º
	sin –ve cos –ve tan +ve	sin –ve cos +ve tan –ve	
	270º	

This can be simplified to show just the positive ratios:

	90º	
	sin +ve	all +ve	
180º			0º, 360º
	tan +ve	cos +ve	
	270º	

Try yourself

- What sign would sin(290º) have?

  negative

- What sign would tan(30º) have?

  positive

- What sign would cos(91º) have?

  negative

- What sign would cos(271º) have?

  positive

Cofunction Identities

$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right) \qquad \sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$$

The Sum and Difference Identities

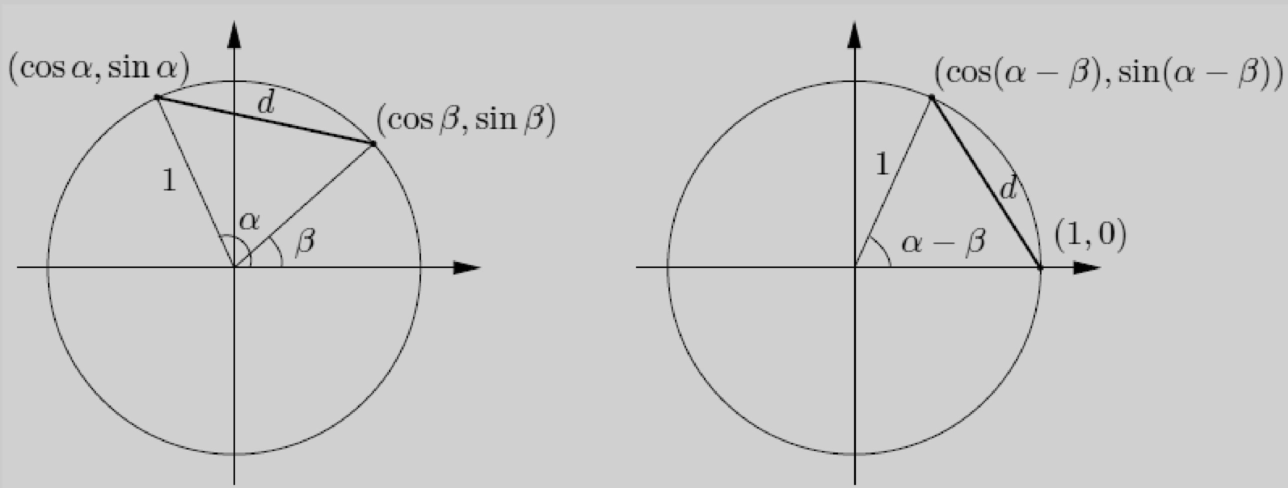
$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

Proving the identity



From the first one we obtain $$d = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}$$

From the second one we obtain $$d = \sqrt{(\cos(\alpha-\beta) - 1)^2 + (\sin(\alpha-\beta) - 0)^2}$$

From these two expressions for d, we can deduce $$d^2 = d^2$$

$$(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = (\cos(\alpha-\beta) - 1)^2 + (\sin(\alpha-\beta) - 0)^2$$

$$\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta = \cos^2(\alpha-\beta) - 2\cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta)$$

$$(\cos^2\alpha + \sin^2\alpha) - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) + (\cos^2\beta + \sin^2\beta) = (\cos^2(\alpha-\beta) + \sin^2(\alpha-\beta)) - 2\cos(\alpha-\beta) + 1$$

$$2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2 - 2\cos(\alpha-\beta)$$

So, we get $$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

Some other important identities

- **Power reduction identities**

$$cos^2(\alpha) = \frac{\cos(2\alpha) + 1}{2} \qquad sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2}$$

For example

*Cos² (30) = (cos (2*30)+1)/2 =(0.5+1)/2=0.75*

- **The double angle Identities**

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$\cos(2\alpha) = cos^2(\alpha) - sin^2(\alpha) = 1 - 2sin^2(\alpha) = 2cos^2(\alpha) - 1$$

For example

*Cos(2*30)=2cos² (30)-1 = 2*0.75-1 =0.5*

Helpful Literature

- D.Lippman, M.Rasmussen (Edition 1.3 2012) *"Precalculus: An Investigation of Functions"*.

THANK YOU

FOR

ATTENTION