Derivatives

Source: Kenneth Kuttler (2009) *Calculus, Applications and Theory*

Definition of Derivative

The derivative of a function is its "rate of change."
At time t, the derivative $\boldsymbol{f'(t)}$ or $\boldsymbol{df/dt}$ or $\boldsymbol{v(t)}$ is

$$\boldsymbol{f'(t) = \lim_{\Delta t \to \infty} \frac{f(t+\Delta t) - f(t)}{\Delta t}}$$

An increasing f (t) has positive slope. A decreasing f (t) has negative slope.

Identities

Derivative of $\frac{1}{t}$ is $\frac{df}{dt} = -\frac{1}{t^2}$ (The function $\frac{1}{t}$ is decreasing, and Δf is below zero. The graph is going downward, and its slope is negative)

Derivative of constant is 0.

$$\frac{d}{dx} cf(x) = c$$

Example - $(5x^2)' = 5 * (x^2)'$

$$\frac{d}{dx}\big(f(x) \pm g(x)\big) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

Example - $(x^2 \pm y^3)' = (x^2)' \pm (y^3)'$

$$\frac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big) g'(x) \quad (\boldsymbol{Chain\ rule})$$

Example – $4(x^3+5)^2 = 4 * 2(x^3+5)(3x^2) = 24x^5 + 120x^2$

$$\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x) \quad (\boldsymbol{Product\ rule})$$

Example –
$\big((5x+x^2)(x^3-2x)\big)' = (5+2x)(x^3-2x) + (5x+x^2)(3x^2-2)$

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \quad (\boldsymbol{Quotient\ rule})$$

Example –

$$\left(\frac{2x+x^4}{4x^2+x^3}\right)' = \frac{(2+4x^3)(4x^2+x^3) - (2x+x^4)(8x+3x^2)}{(4x^2+x^3)^2}$$

	Derivative of $f = \sqrt{x}$ is $f' = \frac{1}{2}x^{-\frac{1}{2}}$ $f(x) = x^n$ $then$ $f'(x) = nx^{n-1}$ $(\boldsymbol{Power\ Rule})$ Example – $(5x^4)' = 5 * 4 * x^{4-1} = 20x^3$ $\frac{d}{dx}\big(\alpha f(x) + \beta g(x)\big) = \alpha f'(x) + \beta g'(x)$ $(\boldsymbol{Linearity\ Rule})$
Marginal Cost and Elasticity in Economics	The ratio $\Delta y/\Delta x$ is the average cost per extra ton. An increase of Δx tons is a relative increase of $\frac{\Delta x}{x}$ A cost increase Δy is a relative increase of $\frac{\Delta y}{y}$ The elasticity of the demand function y(x) is $$E(x) = \lim_{\Delta x \to \infty} \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{dy/dx}{y/x}$$ Elasticity is "marginal" divided by "average." ***E(x)***is also relative change in ***y*** divided by relative change in ***x***
Tangent Line	The curve and its tangent line have the same slope at the crucial point $$y = mx + b \ has\ slope\ m$$ Slope of the tangent line $$\frac{df}{dx} = limit\ of \frac{\Delta f}{\Delta x}$$ The equation of the tangent line has $b = f(a) - ma$: $$y = mx + f(a) - ma \ or\ y - f(a) = m(x - a)$$
Secant Line	through two points. The equation of the curve is still y =f (x). The first point remains at $x = a, y = f(a)$. The other point is at $x = c, y = f(c)$. The secant line goes between them. 1. The slope is $m = \frac{distance\ \ up}{distance\ \ across} = \frac{f(c)-f(a)}{c-a}$ 2. The height is y =f (a) at x = a 3. The height is y =f (c) at x = c The two-point form uses the slope between the points:

	$y - f(a) = \left(\frac{f(c) - f(a)}{c - a}\right)(x - a)$
Common Derivatives	$\frac{d}{dx}(a^x) = a^x \ln(a)$ $\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$ $\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}, x \neq 0$ $\frac{d}{dx}(\log_a(x)) = \frac{1}{xlna}, x > 0$
The Derivatives (second derivative) of the Sine and Cosine and other trigonometric functions	$\frac{d}{dx} sin(x) = cos(x)$ $\frac{d}{dx} cos(x) = -sin(x)$ $\frac{d}{dx} sin^{-1}(x) = \frac{1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} cos^{-1}(x) = -\frac{1}{\sqrt{1 - x^2}}$ $\frac{d^2}{d^2x} \sin(x) = -\sin(x)$ $\frac{d^2}{d^2x} \cos(x) = -\cos(x)$ $\frac{d}{dx} \sinh(x) = \cosh(x)$ $\frac{d}{dx} \cosh(x) = \sinh(x)$ $\frac{d}{dx} tan(x) = sec^2(x)$ $\frac{d}{dx} \cot(x) = -csc^2(x)$ $\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$ $\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$ $\frac{d}{dx} tan^{-1}(x) = \frac{1}{1 + x^2}$ $\frac{d}{dx} cot^{-1}(x) = -\frac{1}{1 + x^2}$ $\frac{d}{dx} sec^{-1}(x) = \frac{1}{x\sqrt{x^2 - 1}}$ $\frac{d}{dx} csc^{-1}(x) = -\frac{1}{x\sqrt{x^2 - 1}}$