Exponential and Logarithmic Functions

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Exponential growth	is a function that grows at a constant percent growth rate $\boldsymbol{f(x) = a(1+r)^x}$ *or* $\boldsymbol{f(x) = ab^x}$ *where b =1+r* ***a*** is the initial or starting value of the function (is the vertical intercept of the graph) ***r*** is the percent growth or decay rate, written as a decimal ***b*** is the growth factor or growth multiplier (determines the rate at which the graph grows the function will increase if $\boldsymbol{b} > 1$ the function will decrease if $\mathbf{0} < b < 1$ The graph will have a horizontal asymptote at $\boldsymbol{y} = \mathbf{0}$ The graph will be concave up if a > 0; concave down if a < 0. The domain of the function is all real numbers The range of the function is $(\mathbf{0}, \infty)$
Transformations of Exponentials	$$\boldsymbol{f(x) = ab^x + c}$$ y = c is the horizontal asymptote
Compound Interest	$$A(t) = a(1+\frac{r}{k})^{kt}$$ ***A(t)*** is the account value ***t*** is measured in years ***a*** is the starting amount of the account, often called the principal ***r*** is the annual percentage rate (APR), also called the nominal rate ***k*** is the number of compounding periods in one year
Annual Percentage Yield	Is the actual percent a quantity increases in one year $$APY = (1+\frac{r}{k})^k - 1$$
Continuous Growth	$$f(x) = ae^{rx}$$ ***a*** is the starting amount ***r*** is the continuous growth rate
Logarithmic Functions	$log_b(x)$ is the inverse of the exponentioal function b^x Graph of Log The graph has a horizontal intercept at $(1, 0)$ The graph has a vertical asymptote at $x = 0$ The graph is increasing and concave down The domain of the function is x > 0, or $(0, \infty)$ The range of the function is all real numbers, or $(-\infty, \infty)$

Properties of Logs	$log_b(b^x) = x$ $b^{log_b x} = x$ $b^a = c <=> log_b(c) = a$ $log_b(A^r) = rlog_b(A)$ $log_b(A) = \frac{log_c(A)}{log_c(b)}$
Inverse Properties	$log_b(b^x) = x$ $b^{log_b x} = x$
Exponential Property	$log_b(A^r) = rlog_b(A)$
Change of Base	$log_b(A) = \frac{log_c(A)}{log_c(b)}$
Sum of Logs Property	$log_b(A) + log_b(C) = log_b(AC)$
Difference of Logs Property	$log_b(A) - log_b(C) = log_b(\frac{A}{C})$
Common Log	is the logarithm with base 10 ($log(x)$)
Natural Log	is the logarithm with base e ($ln(x)$)