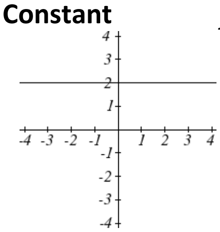
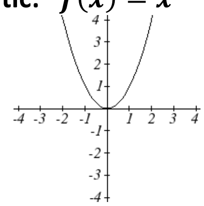
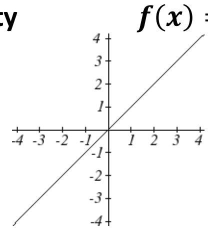
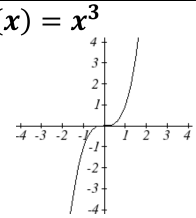
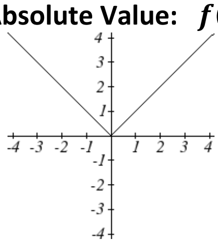
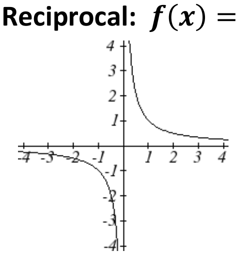
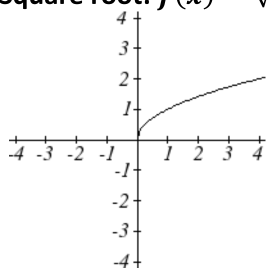
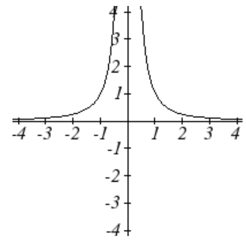
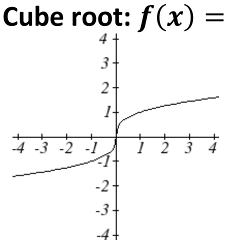
Functions

Source: D.Lippman, M.Rasmussen (2012) *Precalculus: An Investigation of Functions* (Edition 1.3)

Function:	A rule for a relationship between an input, or independent, quantity and an output, or dependent, quantity in which each input value uniquely determines one output value.
Domain:	The set of possible input values to a function. (Input values shown along the horizontal axis of the graph)
Range:	The set of possible output values of a function. (Output values shown along the vertical axis of the graph)

Basic Functions

Constant $\boldsymbol{f(x) = c}$  **Domain:** $(-\infty;\infty)$**, Range:** $[c]$ **Function Increasing/Decreasing –** Neither increasing nor decreasing **Concavity -** Neither concave up nor down	**Quadratic:** $\boldsymbol{f(x) = x^2}$  **Domain**: $(-\infty;\infty)$,**Range:** $[0;\infty)$ **Function Increasing/Decreasing –** Increasing on $(0,\infty)$ Decreasing on $(-\infty,0)$ Minimum at x = 0 **Concavity -** Concave up $(-\infty,\infty)$
Identity $\boldsymbol{f(x) = x}$  **Domain**: $(-\infty;\infty)$, **Range**: $(-\infty;\infty)$ **Function Increasing/Decreasing –** Increasing **Concavity -** Neither concave up nor down	**Cubic:** $\boldsymbol{f(x) = x^3}$  **Domain**: $(-\infty;\infty)$, **Range**: $(-\infty;\infty)$ **Function Increasing/Decreasing** – Increasing **Concavity** - Concave down on $(-\infty,0)$ Concave up on $(0,\infty)$ Inflection point at $(0{,}0)$
Absolute Value: $\boldsymbol{f(x) = \lvert x \rvert}$  **Domain:** $(-\infty;\infty)$, **Range:** $[0;\infty)$ **Function Increasing/Decreasing –** Increasing on $(0,\infty)$ Decreasing on $(-\infty,0)$	**Reciprocal:** $\boldsymbol{f(x) = \frac{1}{x}}$  **Domain:** $(-\infty 0)U(0;\infty)$, **Range:** $(-\infty 0)U(0;\infty)$, **Function Increasing/Decreasing–**Decreasing $(-\infty,0) \cup (0,\infty)$

Concavity - Neither concave up or down	**Concavity -** Concave down on $(-\infty, 0)$ Concave up on $(0, \infty)$
Square root: $f(x) = \sqrt{x}$  **Domain**: $[0; \infty)$, **Range**: $[0; \infty)$ **Function Increasing/Decreasing –** Increasing on $(0, \infty)$ **Concavity -** Concave down on $(0, \infty)$	**Reciprocal squared:** $f(x) = \frac{1}{x^2}$  **Domain:** $(-\infty 0) U (0; \infty)$, **Range:** $(0; \infty)$ **Function Increasing/Decreasing –** Increasing on $(-\infty, 0)$ Decreasing on $(0, \infty)$ **Concavity -** Concave up on $(-\infty, 0) \cup (0, \infty)$
Cube root: $f(x) = \sqrt[3]{x}$  **Domain:** $(-\infty; \infty)$**, Range:** $(-\infty; \infty)$ **Function Increasing/Decreasing –** Increasing **Concavity -** Concave down on $(0, \infty)$ Concave up on $(-\infty, 0)$ Inflection point at (0,0)	

Rate of Change	**describes how the output quantity changes in relation to the input quantity**
Average rate of change	= $\frac{\textit{Change of Output}}{\textit{Change of Input}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
A function is increasing	if the function values increase as the inputs increase. increasing if f(b) > f(a)for any two input values a and b in the interval with b>a. The average rate of change of an increasing function is positive.
A function is decreasing	if the function values decrease as the inputs increase. decreasing if f(b) < f(a)for any two input values a and b in the interval with b>a. The average rate of change of a decreasing function is negative.
Local maximum	A point where a function changes from increasing to decreasing
Local minimum	A point where a function changes from decreasing to increasing
Concave up	if the rate of change is increasing.
Concave down	if the rate of change is decreasing
Inflection point	A point where a function changes from concave up to concave down or vice versa

Composition of functions	the output of one function is used as the input of another
Horizontal Shift	Given a function f(x), if we define a new function g(x) as $g(x) = f(x+k)$, where k is a constant then g(x)is a horizontal shift of the function f(x). If k is positive, then the graph will shift left. If k is negative, then the graph will shift right
Vertical Shift	Given a function f(x), if we define a new function g(x) as g(x)= f(x)+ k, where k is a constant then g(x)is a vertical shift of the function f(x), where all the output values have been increased by k. If k is positive, then the graph will shift up. If k is negative, then the graph will shift down
	Vertical reflection (reflection about the x-axis) – $g(x) = -f(x)$ **Horizontal reflection (reflection about the y-axis) –** $g(x) = f(-x)$
Even function	$f(x) = f(-x)$ (The graph of an even function is symmetric about the vertical axis)
Odd function	$f(x) = -f(-x)$ (The graph of an odd function is symmetric about the origin)
Inverse Function	If $f(a) = b$, then a function g(x)is an inverse of f if $g(b) = a$ $(f-1(x))$

Linear functions

Mathematical modeling	Adding a numerical structure to a real world situation
Linear function	A function whose graph produces a line $\boldsymbol{f(x) = b + mx \text{ or } f(x) = mx + b}$, *m* - constant rate of change (slope) $\boldsymbol{m = \frac{change\ in\ output}{change\ in\ input} = \frac{\Delta y}{\Delta x} = \frac{f(a)-f(b)}{a-b}}$ $\boldsymbol{f(x) = b + mx}$ is an increasing function if $\boldsymbol{m} > 0$ $\boldsymbol{f(x) = b + mx}$ is an decreasing function if $\boldsymbol{m} < 0$ *b* is the vertical intercept
	Horizontal lines have equations of the form $f(x) = b$ Vertical lines have equations of the form $x = a$ Two lines are parallel if the slopes are equal The lines will be perpendicular if the multiply of slopes are equal to -1
Interpolation **Extrapolation**	When we predict a value inside the domain and range of the data When we predict a value outside the domain and range of data
Correlation coefficient -	Value, r, between -1 and 1 r>0 suggests a positive relationship r<0 suggests a negative relationship The closer the value is to 0 the more scattered the data The closer the value is to -1 and 1 the less scattered the data is