Parametric Equations

Usually we write equations in the form $\boldsymbol{y = f(x)}$ or $\boldsymbol{x = g(y)}$, like $\boldsymbol{y = x^2 + 4x + 2}$ or $\boldsymbol{x = \sin y}$.

But we can't use this form to represent certain curves. Some curves can only be represented with **parametric equations**.

When writing parametric equations we include another variable (or **parameter**), usually $\boldsymbol{t}$.

__Example__: $\boldsymbol{x = t^2 - t \quad y = t + 1}$

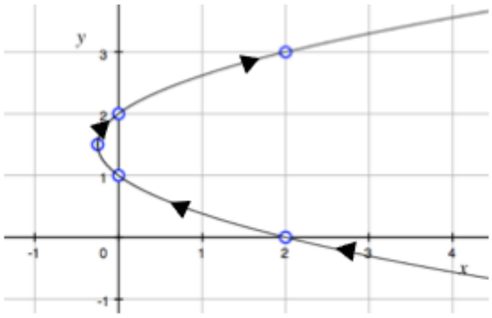
For every value of t, we get a point (x, y). Let's calculate some of the points for this example.

If $t = -1$ → $x = (-1)^2 - (-1) = 2$

$y = (-1) + 1 = 0$

If we continue to calculate values of x and y we can graph the curve as shown below.

t	x	y
-1	**2**	**0**
0	**0**	**1**
0.5	**-0.25**	**1.5**
1	**0**	**2**
2	**2**	**3**



For this example, we can eliminate t and represent the curve with one function in terms of x and y.

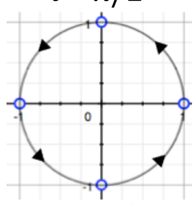
$$x = t^2 - t \qquad y = t + 1$$
$$t = y - 1$$
$$x = (y - 1)^2 - (y - 1)$$
$$x = y^2 - 2y + 1 - y + 1$$
$$x = y^2 - 3y + 2$$

Parametric equations are useful for representing circles or ovals.

__Example:__ $\boldsymbol{x = \cos t \quad y = \sin t}$

If we graph the function we get a circle as shown below.

$t = \pi/2$

$t = \pi$  $t = 0$

$t = 3\pi/2$

In this example we can also eliminate t which will result in the equation for a circle.

$$x = \cos t \qquad y = \sin t$$

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

$x^2 + y^2 = 1$ ← equation for a circle with radius = 1

But what about the direction of the curve?

Before, when learning about curves and functions, we never had to consider the direction of the curve. But in this case, the direction is important because we don't have just any curve, we have a **parametric curve.** So, when graphing parametric curves **look out for how the curve progresses as $\boldsymbol{t}$ increases.**

For further help, check out these resources:

http://www.math.hmc.edu/calculus/tutorials/parametric_eq/

http://www.khanacademy.org/math/trigonometry/parametric_equations/parametric/v/parametric-equations-1

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Stewart, J. (2008). *Calculus: Early Transcendentals*. 6th Ed. Thomson Brooks/Cole.