

Partial Fractions

Example 1: Write $\frac{4x+1}{x^2-x-2}$ using partial fractions.

$$\frac{4x+1}{x^2-x-2} = \frac{4x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} = \frac{A(x-2) + B(x+1)}{(x+1)(x-2)}$$

$$4x+1 = A(x-2) + B(x+1)$$

$$x=2 \Rightarrow 4 \cdot 2 + 1 = A(0) + B(3) \Rightarrow B=3$$

$$x=-1 \Rightarrow 4(-1) + 1 = A(-3) + B(0) \Rightarrow A=1$$

$$\text{Thus } \frac{4x+1}{x^2-x-2} = \frac{1}{x+1} + \frac{3}{x-2}.$$

Example 2: Use partial fractions to find $\int \frac{4x+1}{x^2-x-2} dx$.

$$\begin{aligned} \int \frac{4x+1}{x^2-x-2} dx &= \int \left(\frac{1}{x+1} + \frac{3}{x-2} \right) dx \\ &= \ln(x+1) + 3\ln(x-2) + C \\ &= \ln[(x+1)(x-2)^3] + C \text{ or } \ln(x+1)(x-2)^3 + C \end{aligned}$$