Periodic Functions

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Periodic Functions	A periodic function is a function for which a specific horizontal shift, P, results in the original function: $\boldsymbol{f(x+P)=f(x)}$ for all values of x
Period of Sine and Cosine	The periods of the sine and cosine functions are both 2π.
Negative Angle Identities	The sine is an odd function, symmetric about the origin, so $\sin(-\theta) = -\sin(\theta)$ The cosine is an even function, symmetric about the y-axis, so $\cos(-\theta) = \cos(\theta)$
Midline	The center value of a sinusoidal function, the value that the function oscillates above and below(corresponding to a vertical shift)
Transformations of Sine and Cosine	Given an equation in the form $f(t) = Asin(Bt) + k$ or $f(t) = Acos(Bt) + k$ A is the vertical stretch, and is the amplitude of the function. B is the horizontal stretch/compression, and is related to the period, P, by $P = \frac{2\pi}{B}$ k is the vertical shift and determines the midline of the function.
Horizontal Shifts of Sine and Cosine (phase shift)	Given an equation in the form $f(t) = Asin(B(t-h)) + k$ $or\ f(t) = Acos(B(t-h)) + k$ h is the horizontal shift of the function
The Graph of the tangent function	$m(\theta) = tan(\theta)$
The Period of the tangent function	π
The domain of the tangent function	$\theta \neq \frac{\pi}{2} + k\pi, \text{where k is an integer}$
The range of the tangent function	is all real numbers $(-\infty, \infty)$
The Graph of Secant and Cosecant	The secant and cosecant graphs have period 2π like the sine and cosine functions Secant has domain $\theta \neq \frac{\pi}{2} + k\pi$, where k is an integer Cosecant has domain $\theta \neq k\pi$, where k is an integer Both secant and cosecant have range of $(-\infty, -1] \cup [1, \infty)$
The Graph of Cotangent	The cotangent graph has period π Cotangent has domain $\theta \neq k\pi$, where k is an integer Cotangent has range of all real numbers, $(-\infty, \infty)$

Negative Angle Identities	$\sin(-t) = -\sin(t)$ $cos(-t) = cos(t)$ $tan(-\theta) = -tan(\theta)$ $cot(-\theta) = -cot(\theta)$ $sec(-\theta) = sec(\theta)$ $csc(-\theta) = -csc(\theta$
Inverse Trig Function	if $f(a) = b$, then an inverse function would satisfy $f^{-1}(b) = a$
Inverse Sine, Cosine, and Tangent Functions	For angles in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, if $sin(\theta) = a,$ then $sin^{-1}(a) = \theta$ For angles in the interval $[0, \pi]$, if $cos(\theta) = a$, then $cos^{-1}(a) = \theta$ For angles in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$ if $tan(\theta) = a$, then $tan^{-1}(a) = \theta$
Domain and Range of inverse Trig functions	$sin^{-1}(x)$ has domain $[-1, 1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ $cos^{-1}(x)$ has domain $[-1, 1]$ and range $[0, \pi]$ $tan^{-1}(x)$ has domain of all real numbers and range $(-\frac{\pi}{2}, \frac{\pi}{2})$