Trigonometric Functions of Angles

Source: D.Lippman, M.Rasmussen (2012) *Precalculus: An Investigation of Functions* (Edition 1.3)

Circles :	
The Pythagorean Theorem	the sum of the squares of the legs of a right triangle will equal the square of the hypotenuse of the triangle. $\boldsymbol{a^2 + b^2 = c^2}$
Distance between two points	$(x_1, y_1)\, and\, (x_2, y_2)$ $dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Radius Equation	$\boldsymbol{r = \sqrt{(x-h)^2 + (y-k)^2}}$
Circle Equation	$\boldsymbol{(x-h)^2 + (y-h)^2 = r^2}$
Angles:	
Measure of an angle	is a measurement between two intersecting lines A degree is a measurement of angle.
Coterminal Angles	After completing their full rotation based on the given angle, two angles are coterminal if they terminate in the same position
Arclength	Arclength is the length of an arc, s, along a circle of radius r subtended (drawn out) by an angle θ
Arclength on a Circle	The length of an arc, s, along a circle of radius r subtended by angle θ in radians is $s = r\theta$
Area of a Sector	of a circle with radius r subtended by an angle θ , measured in radians, is $= \frac{1}{2}\theta r^2$
Radians	The radian measure of an angle is the ratio of the length of the circular arc subtended by the angle to the radius of the circle. $radian\ measure = \frac{s\ (length\ of\ an\ arc)}{r\ (\ radius)}$ $1\ degree = \frac{\pi}{180}$ or $1\ radian = \frac{180}{\pi}$
Angular velocity(ω)	point moves along a circle of radius r, can be found as the angular rotation θ per unit time, t. $\omega = \frac{\theta}{t}$
Linear Velocity(v)	of the point can be found as the distance travelled, arclength s, per unit time, t. $v = \frac{s}{t}$
Relationship Between Linear and Angular Velocity	When the angular velocity is measured in radians per unit time, linear velocity and angular velocity are related by the equation v r = ω
The sine function	$\sin(\theta) = \frac{y}{r}$
The cosine function	$\cos(\theta) = \frac{x}{r}$
Domain of Sine and Cosine	is all real numbers, $(-\infty, \infty)$

The range of sine and cosine

is the interval [-1, 1]

Graph

sinusoidal (the shape of the graph begins repeating after 2π)
The slope of the sine curve is given by the cosine curve.
The slope of the cosine curve follows the negative of the sine curve.

Coordinates of the point (x, y)

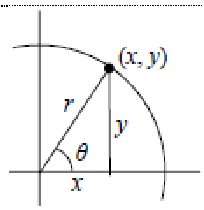
$$x = rcos(\theta)$$
$$y = rsin(\theta)$$
$$cos^2(\theta) + sin^2(\theta) = 1$$

The Pythagorean Identity

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Reference Angle

is the size of the smallest angle to the horizontal axis(between 0 and 90 degrees)



The tangent function: $\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$

The secant function: $\sec(\theta) = \frac{r}{x} = \frac{1}{\cos(\theta)}$

The cosecant function: $\csc(\theta) = \frac{r}{y} = \frac{1}{\sin(\theta)}$

The cotangent function: $\cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$

Identities

$$\frac{1 + \cot(\alpha)}{\csc(\alpha)} = \sin(\alpha) + \cos(\alpha)$$
$$\frac{cos^2(\theta)}{1 + \sin(\theta)} = 1 - \sin(\theta)$$
$$sin^2(\theta) = 1 - cos^2(\theta)$$
$$cos^2(\theta) = 1 - sin^2(\theta)$$

Cofunction Identities

$$\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$$
$$\sin(\theta) = \cos(\frac{\pi}{2} - \theta)$$

The Sum and Difference Identities

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

The Sum to Product Identities

$$\sin(u) + \sin(v) = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin(u) - \sin(v) = 2\sin\left(\frac{u-v}{2}\right)\cos\left(\frac{u+v}{2}\right)$$
$$\cos(u) + \cos(v) = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u+v}{2}\right)$$
$$\cos(u) - \cos(v) = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

The double angle Identities

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$\cos(2\alpha) = cos^2(\alpha) - sin^2(\alpha) = 1 - 2sin^2(\alpha) = 2cos^2(\alpha) - 1$$

Power Reduction Identities

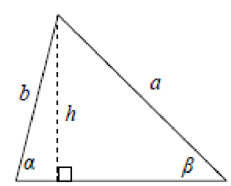
$$cos^2(\alpha) = \frac{\cos(2\alpha) + 1}{2}$$
$$sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2}$$

Half-Angle Identities

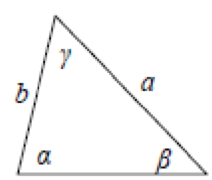
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{\cos(\theta) + 1}{2}}$$
$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

Law of Sines

$$\sin(\alpha) = \frac{h}{b}$$
$$\sin(\beta) = \frac{h}{a}$$

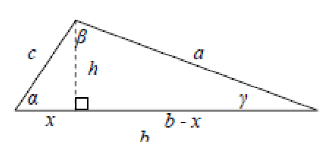


$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c}$$



Law of Cosines

$\cos(\alpha) = \frac{x}{c}$ or $x = c * \cos(\alpha)$



$$a^2 = c^2 + b^2 - 2bc * \cos(\alpha)$$
$$b^2 = a^2 + c^2 - 2ac * \cos(\beta)$$
$$c^2 = a^2 + b^2 - 2ab * \cos(\gamma)$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{tan\frac{1}{2}(\alpha - \beta)}{tan\frac{1}{2}(\alpha + \beta)}$$

	$$\frac{b-c}{b+c}=\frac{tan\frac{1}{2}(\beta-\gamma)}{tan\frac{1}{2}(\beta+\gamma)}$$ $$\frac{a-c}{a+c}=\frac{tan\frac{1}{2}(\alpha-\gamma)}{tan\frac{1}{2}(\alpha+\gamma)}$$